

Validity and reconstructing
arguments (schemes)

Validity

- *Definition:* $X \models A$ if and only if any evaluation satisfying everything in X also satisfies A .
- If the argument from X to A is valid then there is no evaluation making the premises X true and the conclusion A false.
 - **($X \& A$) must be true!**
 - **($X \& \neg A$) must be false!**
- Put the premises of the argument and the *negation of the conclusion* in a list!!

Sometimes it is easy to check validity

P1: When I go to the doctor I always wear black underwear.

P2: I went to the doctor yesterday.

C: I wore black underwear.

P1: When I go to the doctor I always wear black underwear.

P2: I went to the bank yesterday.

C: I wore black underwear.

P1: When I go to the doctor I always wear black underwear.

P2: I wore black underwear yesterday.

C: I went to the doctor yesterday.

P1: When I go to doctor I go to the bank.

P2: I wore black underwear yesterday.

C: I went to the bank yesterday

P1: When I go to the doctor I always wear black underwear.

P2: I did not wear black underwear yesterday.

C: I did not go to the doctor yesterday

Put the premises of the argument and the *negation of the conclusion* in a list!!

- If the propositions cannot be true together, **the argument is valid**;
- If they can be true together, **the argument is not valid**.
- A branch is **closed** when it contains a formula and its negation (the formula need not be atomic). A branch that is not closed is said to be open.

P1: $p \supset q$

$p \supset q$

P2: $r \vee \neg q$

$r \vee \neg q$

C: $((p \vee q) \supset r)$

$((\neg p \vee q) \supset r)$

Commencing
the trees

$$\begin{array}{ll} p \supset q & p \supset q \\ r \vee \neg q & r \vee \neg q \\ ((p \vee q) \supset r) & \neg((p \vee q) \supset r) \end{array}$$

$$\begin{array}{l} p \supset q \\ r \vee \neg q \\ \neg((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \neg r \end{array}$$

$$\begin{array}{l} p \supset q \\ r \vee \neg q \checkmark \\ \neg((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \neg r \\ \swarrow \quad \searrow \\ r \quad \neg q \\ \times \end{array}$$

$$\begin{array}{l} p \supset q \checkmark \\ r \vee \neg q \checkmark \\ \neg((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \neg r \\ \swarrow \quad \searrow \\ r \quad \neg q \\ \times \quad \swarrow \quad \searrow \\ \neg p \quad q \\ \times \end{array}$$

Commencing
the trees

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \\
 \sim((p \wedge r) \supset r)
 \end{array}$$

$$\begin{array}{l}
 p \supset q \checkmark \\
 r \vee \sim q \checkmark \\
 \sim((p \vee q) \supset r) \checkmark
 \end{array}$$

$$\begin{array}{l}
 p \supset q \checkmark \\
 r \vee \sim q \checkmark \\
 \sim((p \vee q) \supset r) \checkmark
 \end{array}$$

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \\
 \sim((p \vee q) \supset r) \checkmark
 \end{array}$$

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \checkmark \\
 \sim((p \vee q) \supset r) \checkmark
 \end{array}$$

$$\begin{array}{l}
 \sim((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q
 \end{array}$$

$$\begin{array}{l}
 p \vee q \checkmark \\
 | \\
 \sim r
 \end{array}$$

$$\begin{array}{l}
 | \\
 p \vee q \\
 \sim r
 \end{array}$$

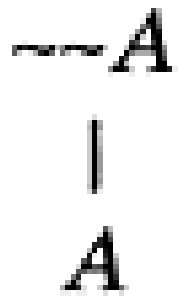
$$\begin{array}{l}
 | \\
 p \vee q \\
 | \\
 \sim r \\
 / \quad \backslash \\
 r \quad \sim q \\
 \times \quad \times
 \end{array}$$

$$\begin{array}{l}
 \sim r \\
 / \quad \backslash \\
 r \quad \sim q \\
 \times \quad / \quad \backslash \\
 \quad \sim p \quad q \\
 \quad \times \quad \times
 \end{array}$$

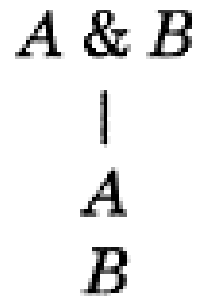
$$\begin{array}{l}
 r \quad \sim q \\
 \times \quad / \quad \backslash \\
 \quad \sim p \quad q \\
 \quad \times \quad \times
 \end{array}$$

Tree rules

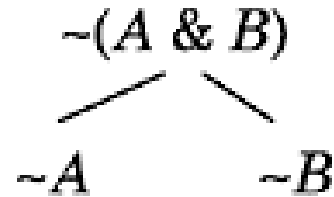
Double negation



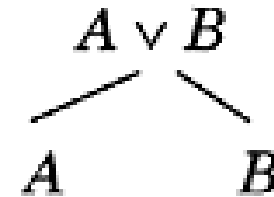
Conjunction



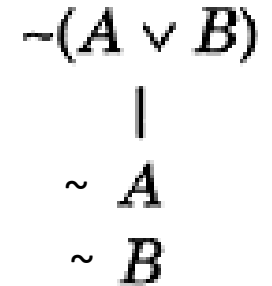
Negated conjunction



Disjunction

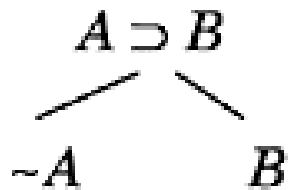


Negated disjunction

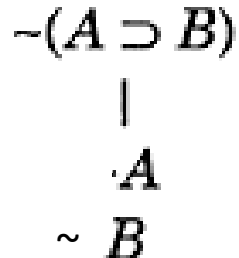


Error corrected

Conditional

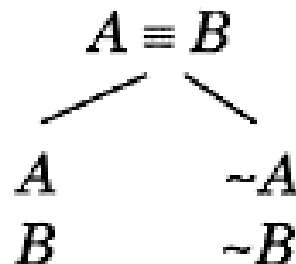


Negated conditional

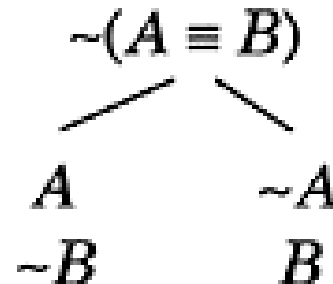


Error corrected

Biconditional



Negated biconditional



Practice in groups!

Create an argument to the form below.

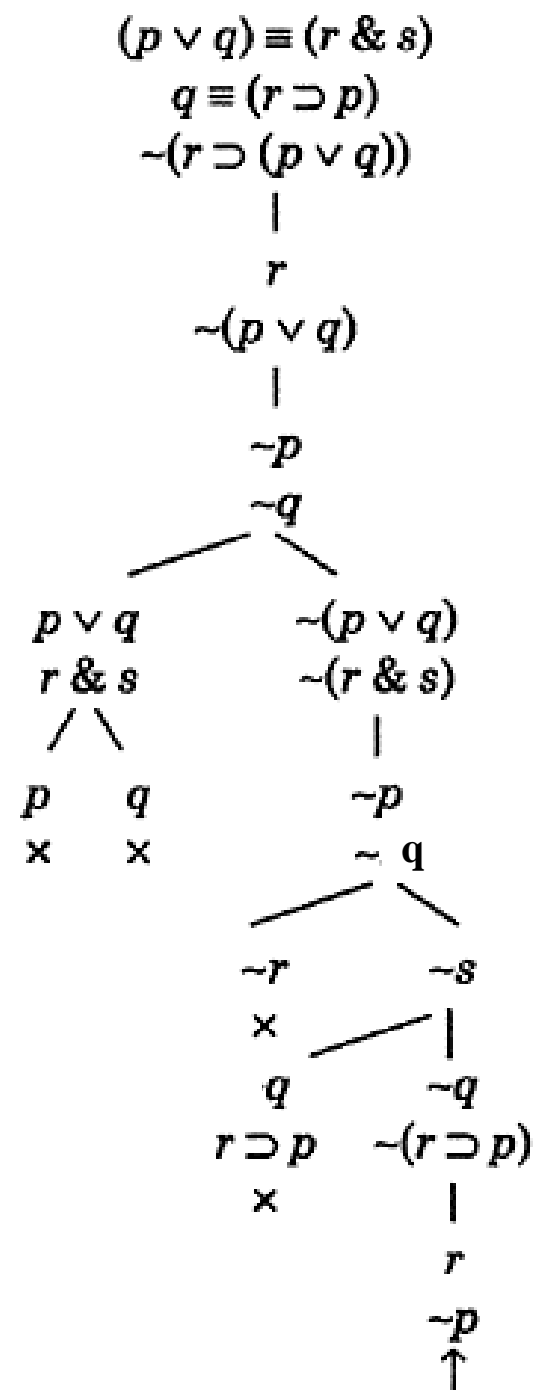
Is this argument valid?

(Error from the book corrected here)

P1: $p \vee q \equiv (r \wedge s)$

P2: $q \equiv (r \supset p)$

C: $r \supset (p \vee q)$



Analyse the following arguments

- All cats are reptiles.
- Bugs Bunny is a cat.
- So Bugs Bunny is a reptile.

- If P, then Q
- P
- Therefore, Q

If P then Q
If Q then S
Therefore if P then S.

- If Socrates was a philosopher, then he wasn't a historian.
- Socrates wasn't a historian.
- Therefore Socrates was a philosopher.

- If P, then Q
- Not Q
- Therefore P

- If P then S
- If Q then R
- Not S or Not R
- Therefore not P or not Q.

A or B
Not A
Therefore B

P or Q
If P then S
If Q then R
Therefore S or R.